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On the initialization of multinomial probit models

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R nlm

Description

This function carries out a minimization of the function `f` using a Newton-type algorithm. See the references for details.

Usage

```
nlm(f, p, ..., hessian = FALSE, typsize = rep(1, length(p)),
    fscale = 1, print.level = 0, ndigit = 12, gradtol = 1e-6,
    stepmax = max(1000 * sqrt(sum((p/typsize)^2)), 1000),
    steptol = 1e-6, iterlim = 100, check.analyticals = TRUE)
```

Arguments

`f`
the function to be minimized, returning a single numeric value. This should be a function with first arg
arguments specified by the ... argument.

If the function value has an attribute called `gradient` or both `gradient` and `hessian` attributes, these wil
Otherwise, numerical derivatives are used. deriv returns a function with suitable `gradient` attribute an

`p`
starting parameter values for the minimization.

?

The multinomial probit model

$$\overbrace{\begin{pmatrix} X_{11} & \cdots & X_{1P} \\ \vdots & \ddots & \vdots \\ X_{J1} & \cdots & X_{JP} \end{pmatrix}}^{X}$$

The multinomial probit model

$$\overbrace{\begin{pmatrix} X_{11} & \dots & X_{1P} \\ \vdots & \ddots & \vdots \\ X_{J1} & \dots & X_{JP} \end{pmatrix}}^{X} \overbrace{\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_P \end{pmatrix}}^{\beta}$$

The multinomial probit model

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The multinomial probit model

$$\overbrace{\begin{pmatrix} U_1 \\ \vdots \\ U_J \end{pmatrix}}^{U} = \overbrace{\begin{pmatrix} X_{11} & \dots & X_{1P} \\ \vdots & \ddots & \vdots \\ X_{J1} & \dots & X_{JP} \end{pmatrix}}^{X} \overbrace{\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_P \end{pmatrix}}^{\beta} + \overbrace{\begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_J \end{pmatrix}}^{\epsilon}$$

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$$y = \arg\max U$$

The multinomial probit model

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$$y = \arg\max U$$

$$\epsilon = L \underbrace{\eta}_{\sim N_J(0,I)} \sim N_J(0, LL' = \Sigma)$$

The multinomial probit model

$$\overbrace{\begin{pmatrix} U_1 \\ \vdots \\ U_J \end{pmatrix}}^{U} = \overbrace{\begin{pmatrix} X_{11} & \dots & X_{1P} \\ \vdots & \ddots & \vdots \\ X_{J1} & \dots & X_{JP} \end{pmatrix}}^{X} \overbrace{\begin{pmatrix} \beta_1 \\ \vdots \\ \beta_P \end{pmatrix}}^{\beta} + \overbrace{\begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_J \end{pmatrix}}^{\epsilon}$$

$$y = \arg\max U$$

$$\epsilon = L \underbrace{\eta}_{\sim N_J(0,I)} \sim N_J(0, LL' = \Sigma)$$

$$\beta = b + O \underbrace{\eta}_{\sim N_P(0,I)} \sim N_P(b, OO' = \Omega)$$

Level normalization

$$\Delta_J U = \Delta_J X\beta + \Delta_J \epsilon, \qquad \Delta_J = \begin{pmatrix} I_{J-1} & -1 \end{pmatrix} \in \mathbb{R}^{(J-1)\times J}$$

$$y = \begin{cases} i, & U_i = \max \Delta_J U > 0, i = 1, \ldots, J-1 \\ J, & \Delta_J U < 0 \end{cases}$$

$$\Delta_J \epsilon \sim N_{J-1}(0, \Delta_J L L' \Delta_J')$$

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Scale normalization

$$(\Delta_J L)_{11} = 1$$

Number of parameters to estimate

alternatives	covariates	$\#b$	$\#O$	$\#\Delta_J L$	total
J	P	P	$P \cdot (P+1)/2$	$(J-1) \cdot J/2 - 1$	
2	2	2	3	0	5
10	10	10	55	44	109

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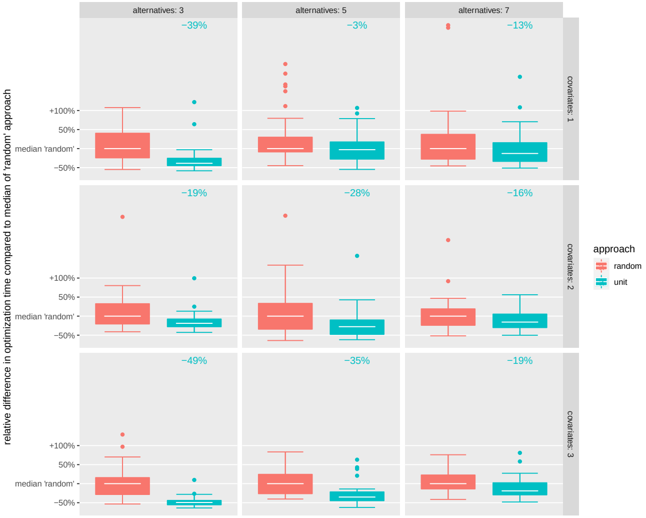
Log-likelihood

$$\log L(y) = \sum_{n,t,j} 1(y_{nt} = j) \overbrace{\Phi_{J-1}(-\Delta_j X_{nt} b \mid 0; \Delta_j (X_{nt} \Omega X_{nt}' + \Sigma) \Delta_j')}^{\Pr(y_{nt}=j)}$$

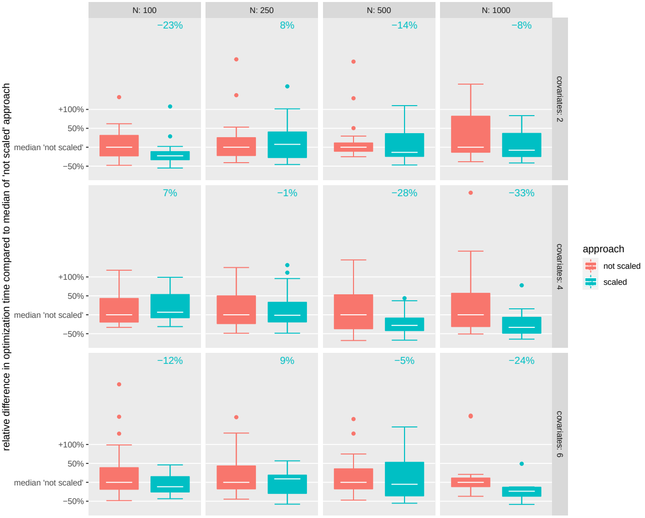
Approach: Unit

- Idea: Choose initial parameters s.t. $b = 0$, $\Omega = I$ and $\Sigma = I$
- Question: Better than random initialization?
- Simulation setting:
  - $N = 50$, $T = 10$
  - random covariates: $1, 2, 3$
  - alternatives: $3, 5, 7$


alternatives: 3
alternatives: 5
alternatives: 7
covariates: 1
covariates: 2
covariates: 3
−39%
−3%
−13%
−19%
−28%
−16%
−49%
−35%
−19%
+100%
50%
median 'random'
−50%
relative difference in optimization time compared to median of 'random' approach
approach
random
unit

Approach: Scaling

- Idea: Different ranges of covariate values may hinder optimization (e.g. 112 minutes travel time, EUR 5 travel cost)
- Question: Does standardization improve optimization time?
- Simulation setting:
  - $T = 10$, alternatives: 3
  - N: 100, 250, 500, 1000
  - random covariates: $2, 4, 6$
  - scale difference: $U[1, 10]$
  - use same random initial guesses, but mind the scales


N: 100
N: 250
N: 500
N: 1000
−23%
8%
−14%
−8%
7%
−1%
−28%
−33%
−12%
9%
−5%
−24%
+100%
50%
median 'not scaled'
−50%
covariates: 2
covariates: 4
covariates: 6
relative difference in optimization time compared to median of 'not scaled' approach
approach
not scaled
scaled

Approach: Subsample

- Idea: Estimate the model on a subsample based on
  - random subsampling (rs)
  - k-means (km)
  - model-based clustering (mc)
- Simulation setting:
  - $T = 10$, alternatives: 3, random covariates: 3
  - N: $100, 250, 500, 1000$
  - subsample proportion: $0.1, 0.25, 0.5$


N: 100
N: 250
N: 500
N: 1000
−7% 24% 3%
−20% −25% −18%
−19% −38% −46%
−69% −63% −58%
−35% −19%
−29% −37% −39%
−30% −22% −49%
−50% −28% −40%
−9% 2% 10%
−2% −5% −34%
−22% 5% −5%
−37% −19% −35%
subsample: 0.1
subsample: 0.25
subsample: 0.5
relative difference in optimization time compared to median of 'full' approach
+100%
50%
median 'full'
−50%
approach
full
km
mc
rs

Approach: Alternating optimization

- Idea: Alternating estimation of parameter groups
  - *b* separately (ao_b)
  - *b* and variances separately (ao_b_var)
  - *b*, variances and covariances separately (ao_full)
- Simulation setting:
  - $N = 50$, $T = 10$
  - random covariates: $1, 2, 3$
  - alternatives: $3, 5, 7$

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Thank you!

Please let me know:

- To what extent is initialization an issue for your models?
- How do you initialize?